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Ruiyao Gao

Technological University Dublin

Aidan O'Dwyer

Technological University Dublin, aidan.odwyer@tudublin.ie

Eugene Coyle

Technological University Dublin

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Model Predictive Control of CSTR Based on Local Model Networks

Ruiyao Gao, Aidan O'Dwyer, Eugene Coyle

*School of Control System & Electrical Engineering
Dublin Institute of Technology,
Kevin St., Dublin 8, Ireland*

Email: Ruiyao.gao@dit.ie, aidan.odwyer@dit.ie, eugene.coyle@dit.ie

Abstract -- A non-linear predictive controller is presented. It judiciously combines predictive controllers with a local model network utilizing a neural-network-like gating system. It avoids the time consuming quadratic optimization calculation, which is normally necessary in non-linear predictive control. A controller simulation on a Continuous Stirred Tank Reactor (CSTR) case study was shown to be satisfactory both in terms of set point tracking and regulation performance over the entire operating range. Moreover, the inherent integration action in the local predictive controller provides zero static offsets.

Key words - model predictive control, local model network, local controller network, non-linearity, neural network.

I. INTRODUCTION

Model predictive control (MPC) techniques have been recognised as efficient approaches to improve operating efficiency and profitability. It has become the accepted standard for complex control problems in the process industries ([1],[2]). It can be used for the control of non-linear systems if they are working around an operating point. However, if the operating point is moved away from the nominal work point, the controller is less effective, or even detrimental to the system operation. One solution to this kind of control problem is to develop a non-linear model predictive control strategy.

Neural networks have been shown to have good approximation capability for non-linear systems. A large number of predictive control schemes have been developed based on Multi-Layer Perceptron (MLP) neural network models since 1990. The key to the successful application of non-linear MPC based on a neural network model is an accurate non-linear model and an efficient optimization algorithm. The back propagation learning algorithm, commonly used in MLP, is essentially a non-linear steepest descent algorithm. It normally involves computationally intensive quadratic optimization and there is no guaranteed global convergence. Furthermore, the network representation is a black box. It fails to exploit the significant theoretical

results available in the conventional modelling and control domain, making it difficult to analyze the behaviour of the controlled system and to prove its stability.

An alternative approach is the use of multiple models, which provide a convenient framework for obtaining both stability and improved performance simultaneously. An interesting approach based on this methodology uses switching, learning and tuning ([3]). This strategy employs different classes of switching and tuning schemes to combine fixed and adaptive models in novel ways. It is particularly suitable for time-varying systems. It shows that an arbitrary switching scheme yields a globally stable system, provided that the interval between successive switches has an arbitrary small but non-zero bound. However, there is no theoretical way to iterate the 'arbitrary small but non-zero bound'. Moreover, this scheme might ask for a large number of local models, especially when the system is highly non-linear.

Johansen and Foss ([4], [5]) introduced local Model Networks (LMN), which are able to use small numbers of locally valid sub-models to approximate a non-linear system across the operating range. At an instant operating point, one dynamic model is formulated by combining the local linear models through a gating system with a neural network structure. The control version of LMN is the Local Controller Network (LCN), which can be formulated instantly through the LMN without extra intense

numerical calculation. The concept was introduced in ([6]) and further extended in ([7]-[8]). Another benefit of using LMN and LCN is that well-developed identification and controller design schemes for linear systems can be properly applied conveniently.

This paper proposes a non-linear predictive controller that performs satisfactorily over the entire operating range of a non-linear system CSTR by a judicious combination of model predictive control and local model networks. The proposed approach was shown to be robust, and to be piecewise linear and continuous, thus reducing the on-line computation to a simple linear function evaluation instead of computationally expensive quadratic optimisation. In addition, the controller contains inherent integral action, which eliminate the static offsets naturally.

The paper is structured as follows. In sections 2 and 3, the construction of local model networks and the model based predictive controller network are discussed. Then the CSTR case study is presented in section 4. Performance results proved the practicality of the method in the modelling and control of non-linear processes.

II. LOCAL MODEL NETWORKS

Local model networks (LMN) were first introduced as a means of decomposing NARMAX models into an insightful structure for system identification and control ([4],[5]). Murray-Smith ([9], [10]) presented further reports on LMN, which put forward this approach as one of the standard techniques to combine linear models and ANN (Artificial Neural Network) to characterise the non-linearity. Figure 1 shows the general structure of this scheme.

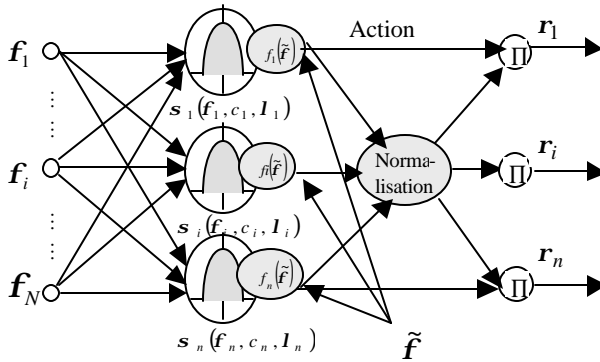


Fig.1. Local Model Networks

We assume that at each time instant, the process behaves in some uniquely characterisable way with each local operating regime f_i , of which we use a

function f_i to describe the property. Then we associate a validity function r_i to determine the validity of the operating regimes given the current operating point \tilde{f} . The modelling problem is to robustly estimate the function f_i from observation data and existing apriori information so as to pre-structure and parameterise the model structure \hat{f}_i . Please refer to ([4] & [5]) for detailed information.

One straightforward and simple approach to the modelling problem is to use a set of linear local models, which is appealing for modelling complex non-linear systems due to its intrinsic simplicity and the weak assumptions required. The linear models can be obtained in several different ways: fitting the parameters of a specified model structure to input/output data obtained from the physical process, fitting the parameters to the simulated response from a detailed fundamental model, or calculating these parameters using differential linearisation.

We shall consider the class of non-linear SISO (single-input single-output) plants expressed in the following operator form

$$\begin{aligned} \dot{x} &= f(x, u) \\ y &= g(x, u) \end{aligned} \quad (1)$$

Linearisation of non-linear dynamic systems of the form of Equation 1, is a standard procedure ([11]). Consider the linearisation of $f(x, u)$ with respect to N designed operating regimes; these linearised models are created and indexed by i together with an operating point vector f and N validity functions, r_i , as follows:

$$\begin{aligned} \hat{f}(x, u) &= \sum_{i=1}^N [A_i(x - x_i^e) + B_i u_i^d] r_i \\ \hat{g}(x, u) &= \sum_{i=1}^N [y_i^e + C_i(x - x_i^e) + D_i u_i^d] r_i \end{aligned} \quad (2)$$

in which, $u_i^d = u_i - u_i^e$ with the superscript e denoting the equilibrium related variable. The state and output of the non-linear system equation (1) can be approximately recreated from the N linear systems of equation (2).

III. MODEL BASED PREDICTIVE CONTROLLER NETWORK

The LCN is the control version of the LMN. In general, the global control signal is defined by

$$u(t) = \sum_{i=1}^{n_f} C_i \left(\mathbf{y}^c(t) \mathbf{r}_i(\tilde{\mathbf{f}}(t)) \right) \quad (3)$$

C_i denotes the local controller for each local model f_i . The n_f local controllers thus obtained are blended using the same validity function, \mathbf{r}_i , which are used in the LMN. The controller information vector \mathbf{y}^c consists of past control inputs, current and past plant outputs, and the current and past values of the reference signal y_{ref} . Figure 2 shows a LCN with a gating system. Its basic idea is to adaptively blend various controllers at different operating regions of the process in a proper way. The gating system \mathbf{r}_i results from the approach formulating the local model network.

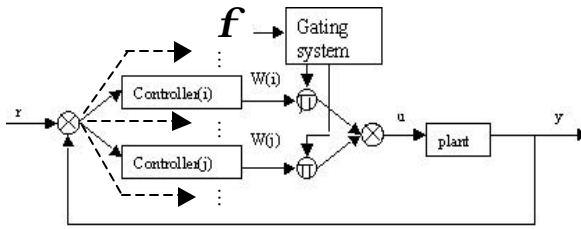


Figure 2. Local controller network

Based on the local linear models, a local model predictive controller, such as GPC (Generalised Predictive Controller) ([12]) can be developed. These local controllers are then combined to make up the LCN. Thus the global controller output is obtained by combining the local controller outputs through the gating system above.

As for the local predictive controller, considering regulation about a particular operating point, a non-linear plant may be generally modelled by a locally linearised CARIMA model (Controlled Auto-regressive and integrated moving average model):

$$A(q^{-1})y(t) = B(q^{-1})u(t-1) + C(q^{-1})\mathbf{x}(t)/\Delta \quad (4)$$

where A and B are polynomials in the backward shift operator q^{-1} , Δ is the differencing operator $1 - q^{-1}$:

$$\begin{aligned} A(q^{-1}) &= 1 + a_1 q^{-1} + \dots + a_{na} q^{-na} \\ B(q^{-1}) &= b_0 + b_1 q^{-1} + \dots + b_{nb} q^{-nb} \end{aligned}$$

If the plant has a non-zero dead-time, the leading elements of the polynomial $B(q^{-1})$ are zero. In equation (4), $u(t)$ is the control input, $y(t)$ is the measured variable or output, and $\mathbf{x}(t)$ is an uncorrelated random sequence. For simplicity, $C(q^{-1})$ is chosen to be 1.

A j -step ahead output prediction on model (4) is given by

$$\hat{y}(t+j) = G_j \Delta u(t+j-1) + f(t+j) \quad (5)$$

Suppose a future set-point or reference sequence $[w(t+j); j=1, 2, \dots]$ is available. The objective of the predictive control law is to drive future plant outputs $y(t+j)$ close to $w(t+j)$ in some sense to minimise a cost function of the form:

$$J(N_1, N_2) = E \left\{ \sum_{j=N_1}^{N_2} [y(t+j) - w(t+j)]^2 + \sum_{j=1}^{N_u} I(j) [\Delta u(t+j-1)]^2 \right\} \quad (6)$$

where N_1 and N_2 are the minimum and the maximum costing horizon, N_u is the Control horizon and $I(j)$ is a control weighting sequence.

The minimisation of J (assuming no constraints on future controls) results in the projected control-increment vector:

$$\tilde{u} = (G^T G + II)^{-1} G^T (w - f) \quad (7)$$

where G is a matrix associated with the local linearised model parameters and f is a function of local linearised model parameters, past control inputs, current and past system outputs ([12]). Note that the first element of \tilde{u} is $\Delta u(t)$, so that the current control $u(t)$ is given by:

$$u(t) = u(t-1) + \bar{g}^T (w - f) \quad (8)$$

where \bar{g}^T is the first row $(G^T G + II)^{-1} G^T$.

Hence, the control includes integral action, which provides zero offset provided that a constant set point $w(t+j) = w$, and for an example, the vector f involves a unit steady-state gain in the feedback path. The control action obtained in equation (8) is seen to contain an integral action that provides zero static offsets. Operational constraints on system input and states can be incorporated into the optimisation procedure in the usual manner. For each of the local models, a local GPC can be constructed and a predictive control action is obtained using equation (8). Then the global control action is formulated through the gating system as described in equation (3). Under the framework of LCN and LMN, some of the stability and robustness analysis for linear model predictive controllers ([13], [14]) could be extended to Local model network based predictive control. These issues are currently under investigation.

IV. CASE STUDY

a) Non-linearity of CSTR

A CSTR (Continuous Stirred Tank Reactor) is a highly non-linear process. A schematic of the CSTR system is shown in Figure 3. A single irreversible, exothermic reaction is assumed to occur in the reactor.

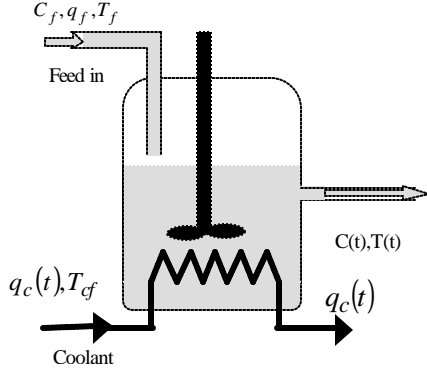


Figure 3. CSTR Plant Model

Table 1. Nominal CSTR Operating Conditions

$q_f = 100$ l/min, product flow rate	$C_f = 1$ mol/l, input concentration
$T_f = 350$ K, input temperature	$T_{cf} = 350$ K, temperature of coolant
$K_1 = 1.44 \times 10^3$ Kl/min/mol,	$V = 100$ l, container volume
$E/R = 104$ K, activation energy	$K_2 = 0.01$ /l, constant
$K_3 = 700$ l/min, constant	$K_0 = 7.2 \times 10^{10}$ min ⁻¹ , constant

The process model consists of two non-linear ordinary differential equations ([15]) as follows.

$$\begin{aligned} \dot{T}(t) &= \frac{q_f}{V} (T_f - T(t)) + K_1 C(t) \exp\left(-\frac{E}{RT(t)}\right) \\ &+ K_2 q_c(t) \left[1 - \exp\left(-\frac{K_3}{q_c(t)}\right)\right] (T_{cf} - T(t)) \\ \dot{C}(t) &= \frac{q_f}{V} (C_f - C(t)) - K_0 C(t) \exp\left(-\frac{E}{RT(t)}\right) \end{aligned}$$

where $q_c(t)$ is the coolant flow rate, $T(t)$ is the temperature of solution and $C(t)$ is the effluent concentration. The model parameters defined, and the nominal operating conditions are shown in table 1. The objective is to control $C(t)$ by manipulating $q_c(t)$. Figure 4 is the locus of equilibrium distribution of input $q_c(t)$ versus output $C(t)$ and $T(t)$; the CSTR exhibits highly non-linear dynamical behaviour. Eigenvalue analysis shows that the stable equilibrium regime of the CSTR lies in

$C(t) \in (0, 0.13566)$ & $q_c(t) \in (0, 110.8)$, which is shown in Figure 5.

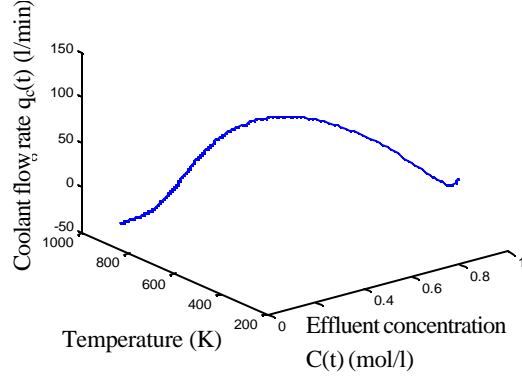


Figure 4: Non-linearity of CSTR

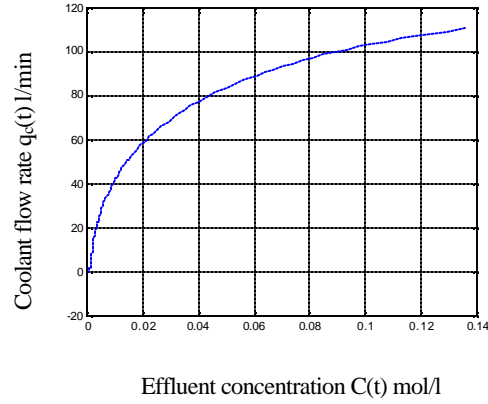


Figure 5. Stable Equilibrium Area

b) CSTR Modelling

The difficulty of modelling using LMN is that it requires careful consideration of the following options: the number of the regimes, the variables to be used to define the regimes, and the size and shape of the regimes. We empirically decomposed the work regime of the CSTR into N small regimes based on a priori information, each of which linearly approximates the local property of the assigned regime. Simulations were carried out to model the system using from 3 up to 10 local models. The global model with 5 local models meets the best trade-off between the number of the local models and the quality of the performance. Simulation results show that over 36 models are needed to get similar performance if the operating regimes are distributed uniformly in an automatic approach.

In this paper, we empirically decomposed the operating area of the CSTR into 5 small regimes based on the study of its non-linearity. The selected operating centre of the 5 local operating regimes are $T(t) = [442, 450, 465, 481, 510]$,

$C(t)=[8.506e-2, 5.8541e-2, 2.9468e-2, 1.4630e-2, 4.5318e-3]$,
 $q_c(t)=[9.8899e1, 8.8291e1, 6.8788e1, 5.0438e1, 2.44335e1]$.

The identification and validation results for the global model are shown in Figure 6, which shows the comparison of the process CSTR output and the model output of one state variable (temperature) $T(t)$ and of the process output variable (effluent concentration) $C(t)$, when the input signal (coolant flow rate) $q_c(t)$ varies from 30 l/m up to 90 l/m, with a 20 l/m interval step. We can see the goodness of the matching between the LMN model and the process CSTR output. They nearly overlap when the control input $q_c(t)$ changes.

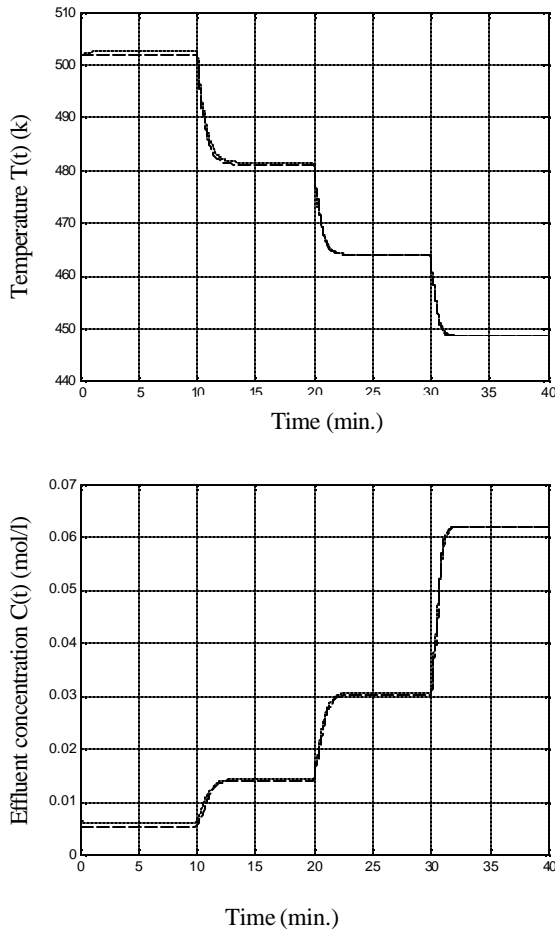


Figure 6: Comparison of the process CSTR output and the LM network output of the temperature $T(t)$ and the effluent concentration $C(t)$, solid line is from the plant, dashed line is from the LMN.

c) Non-linear MPC for CSTR

The global non-linear MPC is formulated by blending the local controllers through the gating system resulting from the LMN structure. It avoids

the computationally intense numerical optimisation, which is a major drawback of normal non-linear predictive controllers.

The set point tracking performance is shown in Figure 7. It is compared with the corresponding outputs from a non-linear PID controller constructed through LMN and LCN approaches ([16]). As displayed, the proposed non-linear MPC presents smooth transient response when the set point $C(t)$ changes between 0.01 and 0.10 and shows better tracking ability globally than the non-linear PID controller does.

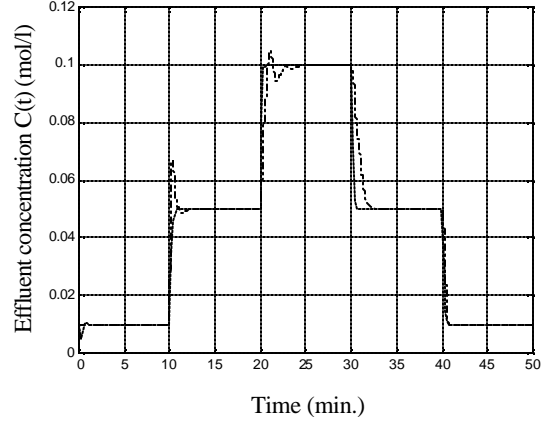


Figure 7: The servo response of the CSTR. The solid line presents the output of the non-linear MPC, the dash-dot line represents the output of the non-linear PID controller.

A simulation is also carried out to examine the regulation performance of the proposed non-linear MPC. It is shown in Figure 8. Introducing an impulse disturbance to the system, CSTR output goes back to the set point after short oscillation under the control of the non-linear MPC. One thing we would like to mention is that the set point $C(t)=0.1$ is very close to the marginal border of the designed controllable region; however, generally, the system output shows sufficient robustness. In contrast, the output from the non-linear PID controller is highly unstable.

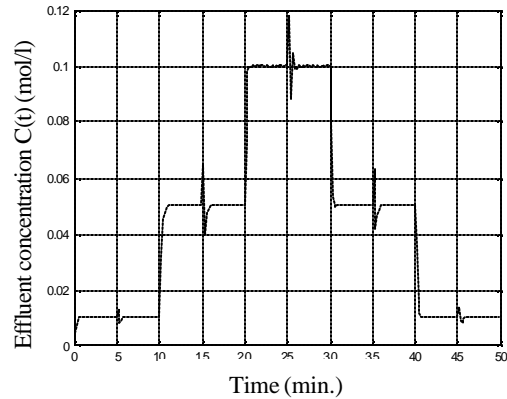


Figure 8. Regulation Performance of CSTR

Moreover, the global performance of the controller highly depends on the performance of the local controllers. The gating system, as a weighting function, smoothes the transient response when the set point changes.

V. RESULTS

We present a non-linear MPC controller using LMN and LCN in this paper. It illustrates the simplicity and practicality of the method in the identification and control of a non-linear system. The simulation highlights the benefits of this scheme for nonlinear system control. The use of LMN enables smooth switching without losing the local meaning and validity of the local controllers. The combination of LCN and MPC enables the global controller to perform perfectly across the entire operating range. Moreover, one advantage of this approach is that it avoids time-consuming numerical optimisation methods and uncertainty in the convergence to the global optimum, which often happen in conventional non-linear model based predictive control.

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